18. SUBSURFACE MULTIPHASE FLOW.

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18.1 Introduction

Chapters 10-13 introduce the differential equation that governs the flow of fluid in porous media. This differential equation is based on Darcy's law and the continuity equation. Multiphase flow in porous media is usually treated by modifying Darcy's law for the reduction in effective permeability when a phase does not completely saturate the pores. For a gas-water system saturated with 50% water and 50% gas, the effective permeability to water might be only 5% of the absolute permeability for a 100% water saturated system, while the gas permeability might be 25% of the absolute permeability.

Multiphase flow in ground water aquifers occurs primarily in the unsaturated zone, i.e. from the ground surface to the water table. In the unsaturated zone both water and gas (which consists mostly air) coexist in the pores of the sediments. For example, it is estimated in chapter 13 that the unsaturated zone in the Gardemoen Project Area is approximately 20% saturated with water. At low water saturations the hydraulic conductivity[a] (i.e. effective permeability) to water is greatly reduced compared with conductivities found in the saturated zone below the water table. This reduction in hydraulic conductivity to water is a primary consideration in subsurface multiphase flow in ground water systems. A secondary factor is the mobility and permanent "loss" of immiscible contaminants such as petroleum products in the form of an immobile residual film.

Other occurrences of multiphase flow in ground water systems include:

(a) three-phase flow when contaminants that enter the unsaturated zone are immiscible (i.e. they do not mix to form a single phase) with the insitu water and gas;

(b) two-phase flow in the saturated zone where an immiscible contaminant such as oil is transported simultaneously with the flowing water; and

(c) two- and three-phase flow during hydrodynamic remediation (cleanup) operations. Once again, the primary concern with multiphase flow in these situations is the relative conductivity of each phase.

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[a] Hydraulic conductivity equals the ratio of permeability to viscosity, $K=k/\mu$; this term is called mobility or transmissibility in the petroleum literature.

The relative permeability (conductivity) of a phase in a multiphase system is a function of several properties, the most important being the saturation of the phase. Other properties affecting relative permeability include pore size distribution, the tendency of a phase to wet the surface of the soil (wettability), and the "history" of saturation change prior to the current conditions (e.g. hysteresis).

For a given geological strata with known pore size distribution, relative permeability is usually considered only a function of saturation. The wetting phase relative permeability is not greatly affected by the flow process (i.e. drainage or imbibition), while the non-wetting phase relative permeability is a strong function of saturation history and hysteresis effects (hysteresis implies a change from drainage to imbibition, or vice versa). In three-phase systems, the intermediate-wetting phase (e.g. oil) must be treated carefully because it is a function of all three phase saturations, and saturation history.

18.2 Relative Permeability Models

Relative permeability of phase p is defined as the ratio of effective permeability of the phase to a base permeability,

$$k_{rp} = \frac{(k_{effective})_p}{k_{base}} \quad (1)$$

The base permeability is usually the permeability of a sample saturated 100% by a liquid, e.g. for water,

$$k_{rw}(S_w) = \frac{k_w(S_w)}{k_w(S_w=1)} \quad (2)$$

However, in petroleum reservoirs the base permeability is usually the effective permeability to oil (or gas) at irreducible water saturation.

Drainage implies that the wetting phase saturation is decreasing. Imbibition implies that the wetting phase saturation is increasing. In ground water systems, drainage flow occurs for example in the lower part of the unsaturated zone when the water table is falling. Imbibition occurs when rain water "soaks" into the unsaturated zone of a ground water system; or in the lower part of the unsaturated zone when the water table is rising.

Two key references on the estimation, measurement, and correlation of relative permeabilities are "Notes on Relative Permeability Relationships" by Standing /1/ and Relative Permeabilities of Petroleum Reservoirs by Honarpour, Koederitz, and Harvey /2/. Standing develops relative permeability relations for two- and three-phase drainage and imbibition flow based on the "capillary" models of Brooks and Corey /3/, Burdine /4/ and others.

Relative permeability estimated based on a capillary model assumes that the porous material behaves like a bundle of capillary tubes of various diameters.[b] The relative permeability and capillary pressure (or capillary retention) curves are closely related through fundamental mathematical relationships (e.g. the Young-Laplace equation relating capillary pressure to mean radius, $P_c=2\sigma/r$, where σ is the interfacial tension and r is the mean pore radius).

An example of relative permeabilities determined from a capillary model is given by Burdine /4/, where he proposes the following relative permeability relationship for the wetting phase in a drainage system,

$$k_{rw} = \frac{k_w(S_w)}{k_w(S_w=1)} = (\hat{S}_w)^2 \frac{\int_0^{\hat{S}_w} \frac{1}{P_c^2} d\hat{S}}{\int_0^1 \frac{1}{P_c^2} d\hat{S}} \tag{3}$$

Other models exist, and the most widely used was presented by Purcell /5/. Capillary models for relative permeability are typically considered adequate for ground water applications.

Honarpour et al. discuss several other relative permeability models based on capillary pressure functions. They also discuss relative permeability models based on statistical, empirical, and network models. These more complicated models are probably not justified in terms of the improved accuracy one might expect, though a "consistent" mathematical model with physical basis would be welcome for the sake of consistency.

For numerical simulation of three-phase systems, the empirical models proposed by Stone /6,7/ for the intermediate-wetting phase (oil) relative permeability have been used extensively in the petroleum industry. Honarpour et al. review three-phase relative permeability measurements and models, including the two Stone models and modifications of these. The estimation of three-phase relative permeabilities is highly uncertain, and particularly for systems exhibiting cycles of hysteresis.

18.3 Estimating Relative Permeability-Saturation Relations

Considering an air-water system where water is the wetting phase undergoing a drainage process, water and gas relative permeabilities (k_{rw} and k_{rg}) can be estimated from the Corey-type /1,2,3/ equations based on the capillary model in Eq. 3,

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[b] The capillary model makes the assumption that there is no interaction between capillaries (pores) of different sizes. That is, only one phase fills any capillary, where the smallest capillaries are filled with the wetting phase and the larger pores are filled with the non-wetting phase.

$$k_{rw} = (\hat{S}_w)^{\frac{2+3\lambda}{\lambda}} \tag{4}$$

$$k_{rg} = (1-\hat{S}_w)^2\left[1-(\hat{S}_w)^{\frac{2+\lambda}{\lambda}}\right] \tag{5}$$

where effective water saturation $\hat{S}_w$ is given by

$$\hat{S}_w = \frac{S_w - S_{wi}}{1 - S_{wi}}$$

S_w is the fraction of pore space filled by water, and S_{wi} is the irreducible water saturation at large capillary pressures. λ is the pore size distribution parameter, where $\lambda \approx 1$ indicates a wide range of pore sizes typical of consolidated sandstones, and $\lambda > 5$ indicates a narrow distribution. Standing mentions that $\lambda=2$ can be used for well-cemented sandstones, $\lambda=4$ for poorly sorted unconsolidated sandstones, and $\lambda=\infty$ for well sorted unconsolidated sandstones. Table 1 /1/ summarizes two-phase drainage relative permeability relations based on the pore size distribution parameter.

Table 1. Two-Phase (Gas-Water) Drainage Relative Permeability Equations (from Standing /1/).

Porous Media	Dist. Factor λ	Wetting Phase (Water) k_{rw}	Non-Wetting Phase (Gas) k_{rg}
Very wide range of pore size	0.5	$(\hat{S}_w)^7$	$(1-\hat{S}_w)^2[1-(\hat{S}_w)^5]$
Wide range of pore size	2	$(\hat{S}_w)^4$	$(1-\hat{S}_w)^2[1-(\hat{S}_w)^2]$
Medium Range of pore size	4	$(\hat{S}_w)^{3.5}$	$(1-\hat{S}_w)^2[1-(\hat{S}_w)^{1.5}]$
Uniform pore size	∞	$(\hat{S}_w)^3$	$(1-\hat{S}_w)^3$

Note: Relative permeabilities are defined for a gas-water system where water is the wetting phase. Also, base permeabilities are:

$$k_{rw}=k_w/(k_w)_{\hat{S}_w=1} \text{ and } k_{rg}=k_g/(k_g)_{\hat{S}_w=0}$$

Usually, however, a multipler k^o_{rg} should be used to correct the gas relative permeability, k^o_{rg} is the relative permeability at irreducible water saturation.

The pore size distribution factor (λ) can be determined experimentally from a measured capillary pressure curve[c], $P_c(S_w)$, where

$$P_c = P_{ce} \cdot (\hat{S}_w)^{-1/\lambda} \tag{7}$$

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[c] Equivalently, a capillary retention curve, $h(S_w)$, can be used to determine λ. h is the height above the zero capillary pressure level, where h is related to capillary pressure by the relation $P_{cgw}=P_g-P_w=(\varrho_w-\varrho_g)(g/g_c)h$.

Plotting log P_c versus log $\hat{S}_w$ should result in an approximately linear trend, where the slope equals $-1/\lambda$. The intercept at $\hat{S}_w=1$ gives P_{ce}, the entry capillary pressure required before the non-wetting phase (gas) will enter and displace the wetting phase (water) from a 100% saturated sample.

Drainage and imbibition relative permeabilities of the wetting phase are usually assumed to be equal. The non-wetting phase (gas) relative permeability is strongly dependent on the direction of saturation change and hysteresis effects. However, k_{rg} is not usually important in ground water applications, except perhaps in remediation by air sparging (see chapter 20, pg. 20-7).

18.4 References

1. Standing, M.B.: "Notes on Relative Permeability Relationships," Dept. of Petroleum Engineering and Applied Geophysics, Norwegian Institute of Technology (NTH), Trondheim (1974).

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7. Stone, H.L.: "Estimation of Three-Phase Relative Permeability and Residual Oil Data," J.Can.Pet.Tech. (1973)12, 53.