

Well Flow Equations (Rate)

Assumption: "Pseudo Steady State"
"Steady State"

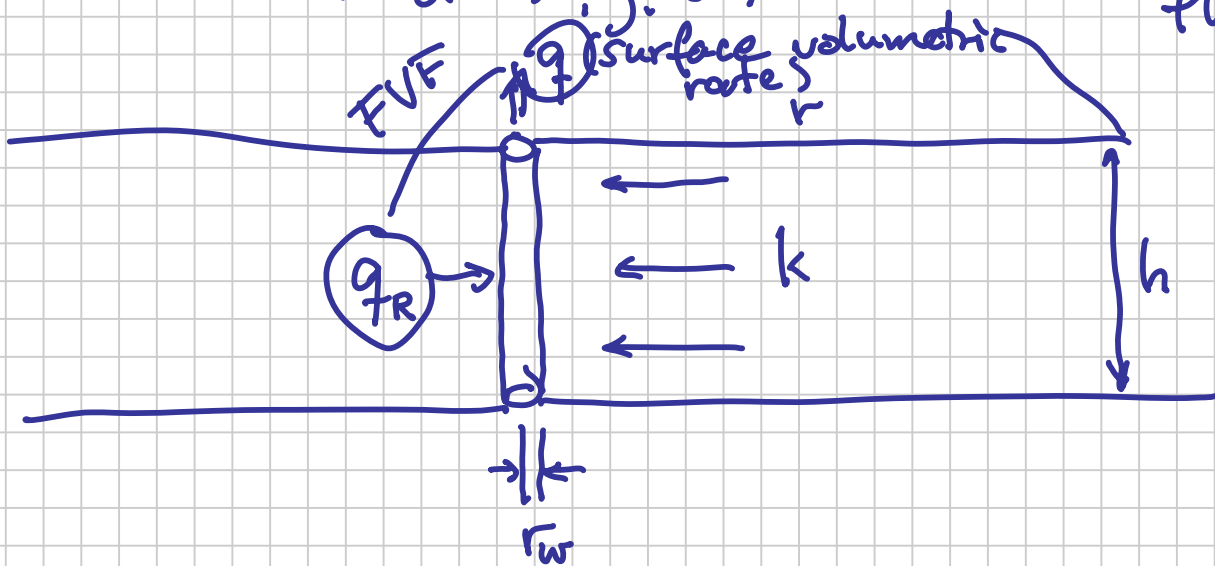
Darcy's Law:

$$v = \frac{k}{\mu} \cdot \frac{dp}{dr} \quad (\text{"Darcy Velocity"})$$

$k = \text{absolute perm. [L}^2\text{]}$ md, D
 $\mu = \text{dynamic viscosity [mPa}\cdot\text{s]}$ 10^{-3}D $\sim 10^{-12} \frac{\text{m}^2}{\mu\text{m}^2}$

$p : \left[\phi \equiv p + \rho g z \right]$

$r : \text{ or } x, y, z ;$



wellbore radius $\sim 0.1 \text{ m}$

$$v = \frac{k}{\mu} \frac{dp}{dr}$$

B.C.

$$p = p_e \text{ @ } r = r_e \quad (\text{Const. outer boundary}) \quad \Rightarrow \text{Steady State}$$

$$p = p_{wf} \text{ @ } r = r_w$$

Assume:

$$\mu, k \sim \text{constant}$$

Cylindrical, Radial Flow Geometry

$$\Rightarrow q_r = 2\pi r h \cdot v$$

$$B \equiv FVF = \frac{q_r}{q} \leftarrow \text{Surface vol. rate}$$

$$q = \frac{q_r}{B} = \frac{2\pi r h v}{B}$$

$$B(p) : B_g \propto \frac{1}{p}$$

$B_o \sim \text{kinda const.} \quad \parallel \text{ Liquid Flow}$

$$q_o = 2\pi h \cdot \frac{r}{B_o} \cdot \frac{k}{\mu} \cdot \frac{dp}{dr}$$

SS Assumption: $q(r) = \text{constant}$

$$\int_{r_w}^{r_e} \frac{1}{r} dr = \frac{2\pi kh}{q_0 \mu_0 B_0} \int_{P_w}^{P_e} dp$$

~ const
for
liquid
flow

$$\ln \frac{r_e}{r_w} = \frac{2\pi kh}{q_0 \mu_0 B_0} (P_e - P_{wf})$$

$$q_0 = \frac{2\pi kh (P_e - P_{wf})}{\mu_0 B_0 \cdot \ln \frac{r_e}{r_w}}$$

	min	Range	max	Unit	O.M. = $\log \left(\frac{\max}{\min} \right)$
① k	0.001		10 000	md	7
h	1		1000	m	3
kh	0.01		10^6	md-m	8
P_e	10		1000	bara	2
P_{wf}	1		100	bara	2
$(P_e - P_{wf})$	10		1000	bar	2
μ_0^*	0.1		10 000	cp	5
B_0	1.		3	vol/vol	0.5
$\ln \frac{r_e}{r_w}$	5		8		0.0x
r_e	100		10 000	m	

* Norway : 0.1 - 10 cp

Non-Ideal (Real Life) Situations:

- ✓ ① Pressure dependant μ, B $\mu(p), B(p)$
- ✓ ② Two-phases flowing (e.g. g+o; o+w; g+w)
 k_{rp}
- ✓ ③ Irregular Well Geometries, s_G
- ✓ ④ $P_e \neq \text{constant}$, i.e., depletion ($P_e(t)$)
- ✓ ⑤ Near wellbore "damage" (Skin Effect)
- ✓ ⑥ Darcy's Law too optimistic:
"turbulent" flow at high velocities
$$\frac{dp}{dr} = \frac{\mu}{k} v + \beta \rho v^2$$

Darcy "Laminar" non-Darcy "Turbulent"

rock property density
- ✓ ⑦ Low-permeability (low-diffusivity; $\frac{k}{\phi \mu c_f}$)
Steady-state (Pseudo Steady State) assumption is not valid.
"system" (rock+fluid) isothermal compress.

Diffusivity Eq.

$$p(x, y, z, t) \text{ for } q = \text{constant}$$

$$\dots v(x, y, z, t) = \frac{k}{\mu} \frac{dp}{dx}$$

2 terms:

(1) $\frac{kh}{\mu B}$

(2) $\frac{k}{\phi \mu c_t} \cdot z$

$$q_p = \frac{kh}{[X + s]} \int_{p_{wf}}^{\bar{p}_r(t)} \frac{k r_p}{\mu_p B_p} dp$$

$\bar{p}_r(Q)$ Depletion
 \uparrow
 Cum. Prod. = $\int q dt$

Usually: $p_e \approx$ volumetric average pressure $\approx \bar{p}_r$
 because using \bar{p}_r instead of p_e

Ideal: $X = \ln(r_e/r_w) - 0.5$

$s = \text{skin} = 0$

$k r_p = 1$

$\mu_p, B_p \approx \text{constant}$

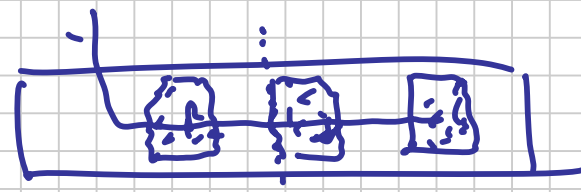
Well Geometry:

$$\underbrace{\ln \frac{r_e}{r_w}}_8 + s$$

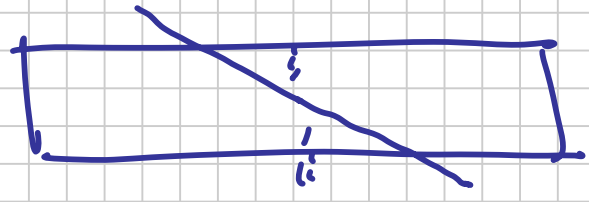
S

$$S = S_G$$

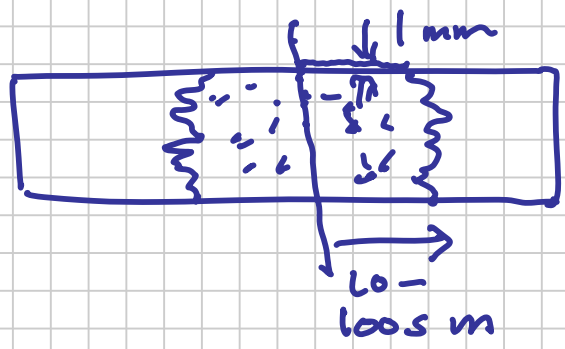
-4 to -7 • Horizontal Well



-1 to -3 • Inclined Well

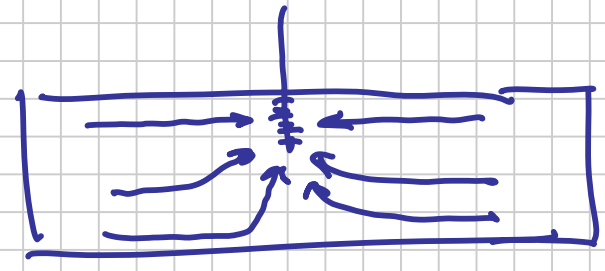


-3 to -5 • Stimulated, Fractured



+2 to +20

• Partial Penetration



Water

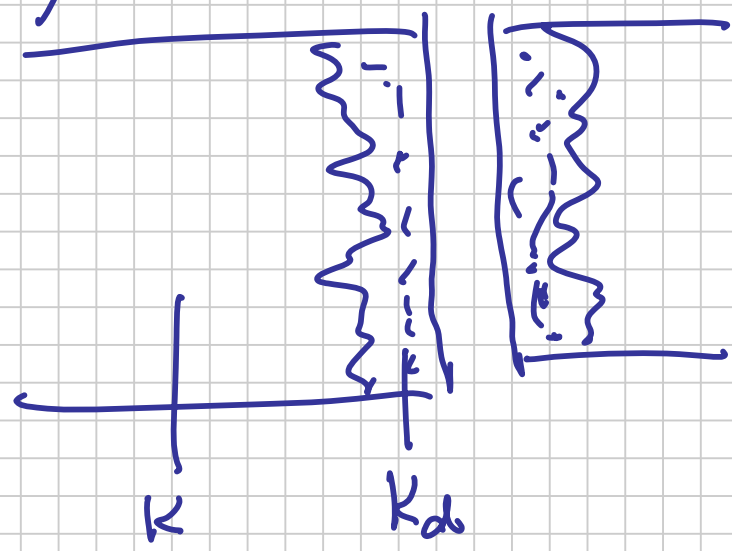
+1 to +100

Damage:

$$S = S_d \sim \left(\frac{k}{k_d} - 1 \right) \ln \frac{r_d}{r_w}$$

0.8 to 0.01

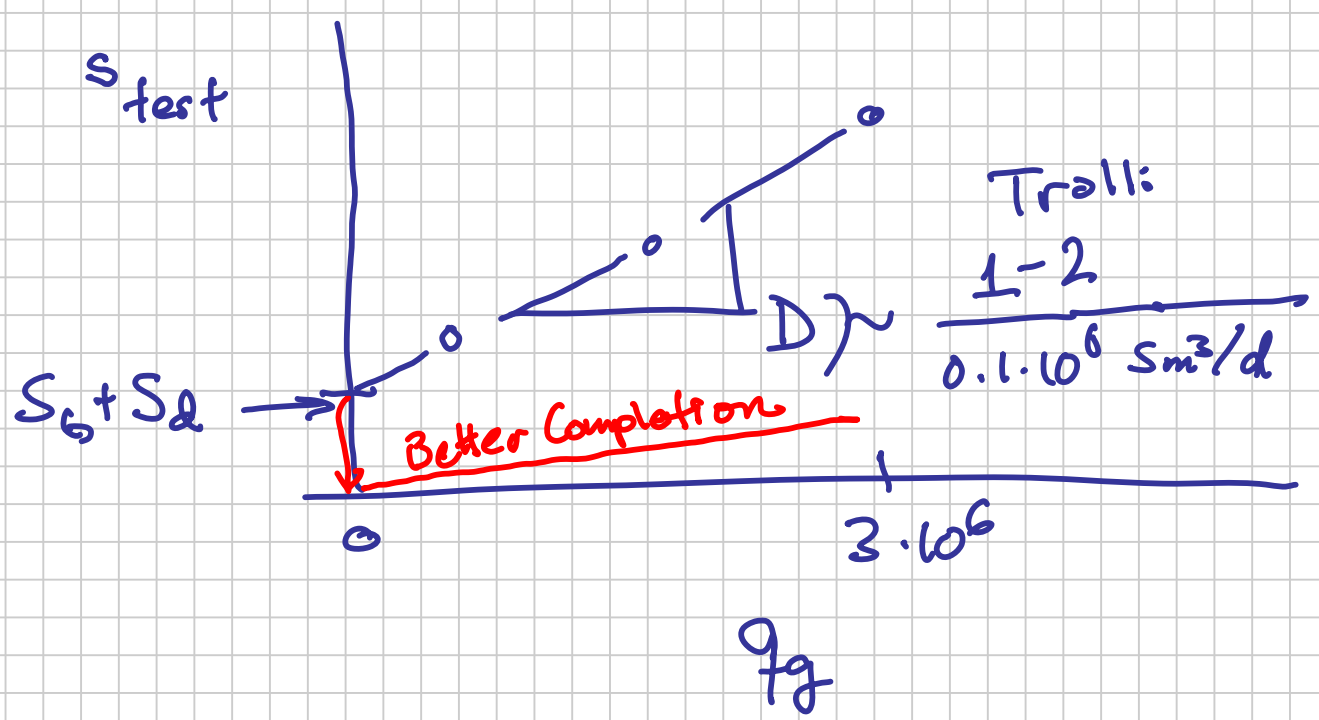
0.1 - 1 m



Non-Darcy Flow (Forchheimer Eq.)

$$+ \beta \rho v^2$$

$$S = S_a + S_d + \underbrace{D \cdot q}_{\substack{\text{Constant} \\ (\beta)}}$$



Low- k , "Transient"
("Infinite-Acting")

$$X = p_D(t_0)$$

dimensionless "pressure drop"

P_D

