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A Treatment of the Gas Percolation Problem in Simulation of Three-Dimensional, Three-Phase Flow in Reservoirs

K. H. COATS
MEMBER AIME

THE U. OF TEXAS
AUSTIN, TEX.

ABSTRACT

This paper describes an approximate technique for handling the problem of percolation of evolved gas upwards through the oil column in computer simulation of natural depletion. This technique has been incorporated into a general model for simulating three-phase flow in one, two or three dimensions. The mathematical model for performing these calculations is described in detail in a separate article.[1]

While vertical gas percolation occurs during natural depletion in a reservoir of any configuration, it is especially pronounced in the pinnacle reef or bioherm. The reef may have an areal extent less than one section with a thickness of up to 800 ft. The upward percolation of evolved gas often limits the time step in calculations as little as one day. Using this small time step, computer expense for a single 30- to 40-year simulation on even a one-dimensional basis has exceeded $2,000.

The method described here for handling the percolation allows time steps of 60 days or more (depending on reservoir size and production rate), resulting in a considerable reduction in computing expense. The method also allows calculations in which secondary gas caps build up in tight zones of the oil column below the main gas cap. The validity of the method is indicated in connection with an example pinnacle reef field. Calculated results using a 2-day time step (where the method in question is not needed and not invoked) are compared with results using the method and a 60-day time step. The comparison shows good agreement. Results from one- and two-dimensional simulations of the reef are presented along with the corresponding computing times on the CDC 6600 computer. A three-dimensional simulation was also performed and the required computing time is given.

INTRODUCTION

A general analysis for simulating three-dimensional, three-phase flow in reservoirs has been developed.[1] In applying this model to reservoirs undergoing natural depletion, a time-step restriction was encountered due to the flow of evolved gas upwards through the oil column and toward the gas cap. A remedy for this problem has been incorporated in the analysis and is described here.

The time-step restriction is encountered only in calculations which include flow in the vertical or near vertical direction. The restriction occurs to some extent in reservoirs of any geometrical configuration, but it is especially pronounced in the pinnacle reef or bioherm where the ratio of thickness to areal extent is unusually large. This paper describes the problem, a method of handling it, incorporation of the method in the three-dimensional, three-phase model, and, finally, a test of the method's validity in an application to an example pinnacle reef reservoir.

THE PROBLEM

During the early stages of reservoir depletion, pressure falls below bubble point in progressively lower regions of the oil column. Continuing production results in evolution of dissolved gas throughout the oil column. This gas then percolates upwards toward the top of the reservoir. If the gas encounters a sufficiently low permeability zone in its travel upward, then it can accumulate and form a secondary gas cap. If no zones are encountered which are tight enough to hold gas against the gravity forces, then the gas travels on until it reaches the top of the sand or the main gas cap.

Numerical simulation of multiphase flow in reservoirs is generally performed with time steps such that the flow of a fluid into or out of a block in one time step is a fraction (considerably less than one) of the total amount of that fluid present in the block. Use of a time step where the time step's

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[1] References given at end of paper.

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flow is a multiple of the amount of gas in place in place in a block often results in severe computational difficulties. The gas percolation during natural depletion causes computational difficulty because of the large ratio of vertical gas flow rate (Mcf/D) through a block of the reservoir to the gas present (Mcf) in the block. This problem is illustrated by Fig. 1, which shows a column of blocks representing a portion of the grid system employed in numerical solution of the reservoir fluid-flow equations. The blocks are 10 ft thick with 15 md permeability and 15 percent porosity. The column is essentially oil saturated with a constant rate of gas injection at the bottom. Oil and gas specific weights expressed as psi/ft are 0.3 and 0.06, respectively, and gas viscosity is 0.015 cp. For the case where vertical viscous forces are small compared to the gravitational gradient, the pressure gradient is 0.3 psi/ft, or approximately that of oil. By Darcy's law the gas-flow rate vertically upwards is

$$q_g = b_g \frac{k\, k_{rg}}{\mu_g} \left(\frac{\partial p}{\partial z} - \gamma_g\right) \quad \frac{\text{Mcf}}{\text{sq ft-day}}$$

where capillary forces are ignored and $p_g = p_o = p$.

The gas in place in one of the blocks is $\phi \Delta z\, b_g\, S_g$ Mcf/sq ft of the block area normal to the z direction. Thus, in a time step Δt, the ratio of Mcf flowing through the block to Mcf contained in the block is

$$\frac{k\, k_{rg} \left(\frac{\partial p}{\partial z} - \gamma_g\right) \Delta t}{S_g\, \phi\, \mu_g\, \Delta z}.$$

Approximating k_{rg} by $.5\, S_g$ for small gas saturation and inserting values for k, Δz, etc., gives this ratio as

$$\frac{(.00633)\ 15\ (.5)\ (.3 - .06)\ \Delta t}{.15\ (.015)\ (10)}$$

$$= .5\ \Delta t.$$

Thus a time step of about 2 days will result in an amount of gas flowing through a block in a single time step that is equal to the entire gas in place in the block. A 60-day time step would require the calculations to handle in one time step a gas flow equal to 30 times the block's content.

AN ASSUMPTION

Calculated saturation distributions in three-phase simulations of a large number of natural depletion cases exhibited a characteristic which is perhaps intuitively obvious. This characteristic was a semi-stabilized gas saturation distribution in the oil column throughout which the evolved gas was percolating upwards. As production rate changed or, in early stages, as additional reservoir volume passed below bubble point, this gas saturation distribution would readjust. The distribution would readjust rapidly, however, after a rate change and then remain nearly unchanged (stabilized) as long as the production rate remained about the same. The intuitive aspect of this characteristic is that a semisteady state should be expected to develop quickly in a situation where the gas flow through a block over a few days time is several times the amount of gas in the block.

The assumption posed to handle the gas percolation problem is: If, under the prevailing calculated pressure gradient, a block in the oil column will flow as much or more gas vertically in a time step as is in the block, then the gas saturation in that block is assumed to be stabilized. If, however, the block will flow less than its content, then the assumption is not invoked. Thus, no modification in the model calculations is made for blocks in sufficiently low permeabiiity regions or in regions where viscous forces are sufficiently large that the block will flow less gas vertically than its content.

A TEST FOR INVOKING THE ASSUMPTION

At the beginning of each time step each block in the oil column is tested to determine whether it will flow more gas vertically than it contains. Consider the two Blocks k-1 and k in a vertical column at some x-y areal position, as shown in Fig. 1. The previous time-step's calculations give water, oil and gas potentials in each of these blocks at the present time t. In the coming time-step Δt, the Mcf of gas flowing from Block k to k-1 can be estimated as

$$q_k = \left(\frac{kA}{L} \frac{k_{rg}\, b_g\, \gamma_g}{\mu_g}\right)_{k-\frac{1}{2}} (\Phi_{gk} - \Phi_{gk-1})\ \Delta t, \quad \ldots \quad (1)$$

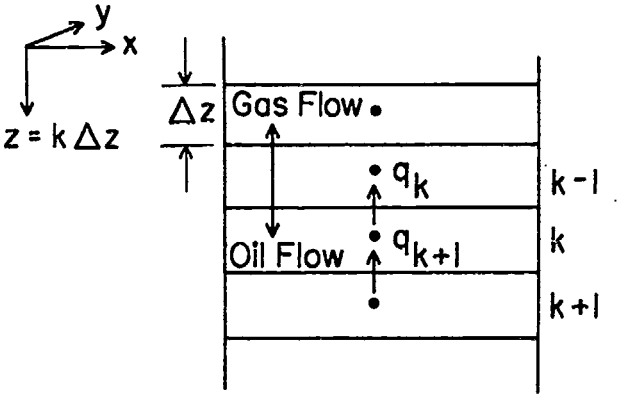

FIG. 1 — UPWARD GAS FLOW IN A COLUMN OF BLOCKS.

where A is the cross-sectional area for flow between Blocks k-1 and k, and L is the distance between block centers. The amount of gas in the block is $V_p b_g S_g$ where V_p is block pore volume. The ratio of flow to gas content is

$$r = \left(\frac{kA}{L}\right)_{k-\frac{1}{2}} \frac{m \gamma_g (\Phi_{gk} - \Phi_{gk-1}) \Delta t}{\mu_g V_p} \quad . . (2)$$

where k_{rg} has been replaced by mS_g. The term m is the slope of the gas relative permeability curve at low gas saturation. If the ratio is greater than one for a block, then the assumption of a steady or semisteady-state gas saturation in that block is made in the coming time step. If the ratio is less than one, then the assumption is not made.

INCORPORATING THE ASSUMPTION INTO THE FLOW CALCULATIONS

For illustration, we will consider a column of blocks at time t at any areal $(x\text{-}y)$ position in a three-dimensional grid comprising the reservoir (see Fig. 1). To simplify illustration of the use of the assumption in the three-phase flow model, we will assume that at the current time t the ratio of Eq. 2 is greater than 1 for all Blocks k = 1,2, . . ., K. The calculation described now for a general Block k is begun with the bottom Block K and repeated in order for Blocks K-1, K-2, ... The calculation for each Block k results in an estimate of the flow of gas, q_k Mcf, out of Block k to Block k-1 over the coming time step Δt. Thus, in illustrating the calculation for Block k we take the flow q_{k+1} from Block k+1 to Block k as a known quantity.

In the coming time step Δt, the approximate gas flow upwards out of Block k to Block k-1 is given by Eq. 1. For a three-dimensional calculation, the flow into the block from the four adjacent blocks in the same horizontal plane (k) can be estimated by summing the products of interblock gas transmissibilities and gas-flow potential differences. This net flow of gas will be denoted by q_{xy} Mcf. The gas saturation S_g in Block k at time t is known from the previous time-step's calculations. If S_{gf} denotes the gas saturation at the end of the coming time step, then a material balance (in – out = accumulation) gives

$$q_{xy} + q_{k+1} - \left(\frac{kA}{L} \frac{k_{rg} b_g \gamma_g}{\mu_g}\right)_{k-\frac{1}{2}} (\Phi_{gk} - \Phi_{gk-1}) \Delta t = V_p b_g (S_{gf} - S_g)$$

The relative permeability is represented as $m(S_{gf} - S_{gc})$ where S_{gc} is critical gas saturation. If all pressure and saturation dependent quantities in this equation are taken at time t, then they are known and the single unknown is S_{gf}. The equation is solved for S_{gf} and the flow q_k is then calculated from Eq. 1 using $k_{rg} = m(S_{gf} - S_{gc})$. This calculation is then repeated for Block k-1, etc. If S_{gf} is less than S_{gc}, q_k is then set to zero; i.e., the flow rate from Block k to k-1 is zero.

After performing the calculations for all blocks we have estimates of gas flow q_k from Block k to k-1 for k = K, K-1, K-2, . . ., 2. In the coming time step, then, the *net* gas flow $q_{k+1} - q_k$ into each block is inserted as part of the source term in the flow calculation (the term normally used to represent injection or production for a well), and the gas-phase interblock transmissibilities are set to zero.

This illustration assumed that all blocks would flow more gas vertically than they hold. Actually, reservoir heterogeneity results in the ability of certain blocks to hold more than they will flow in a time step. For such a Block k the gas transmissibility at k–½ is not set to zero and gas flow between Blocks k and k–1 is calculated normally in the coming time-step's calculation. The flow q_{k+1} to this block from Block k+1, estimated as described above, is entered as the source term for Block k in the coming time-step's calculation.

This method of handling the gas percolation has several satisfying characteristics. First, it is selectively applied only to those blocks where the gas flow/content ratio is high. These blocks, by virtue of their high ratio, generally satisfy the assumption of stabilized gas saturation distribution inherent in the method. Second, the use of the method is automatic; i.e., the test for using or not using it on a given block is simple and easily programmed. Finally, the method involves no forced specification of gas saturation. That is, only the gas-flow rates in the stabilized zones of the oil column are specified; the saturations in the blocks are freely calculated as part of the general three-dimensional, three-phase calculations.

TESTING THE ASSUMPTION'S VALIDITY

Fig. 2 is a sk. of an example pinnacle reef. Porosity and cross-sectional area normal to the z direction are given for each 10-ft thick layer in the top 360 ft of pay. Fig. 3 is a bar chart of permeability vs depth and shows the pronounced heterogeneity (note the logarithmic scale on permeability). Table 1 gives relative permeabilities and capillary pressures as functions of saturations, and Table 2 tabulates the pressure dependent quantities. Simple, linear capillary pressure curves were employed because capillary forces negligibly affected calculated performance for all cases considered. Table 3 completes the data employed in the calculations, including the production rate of 760 STB/D. The reservoir

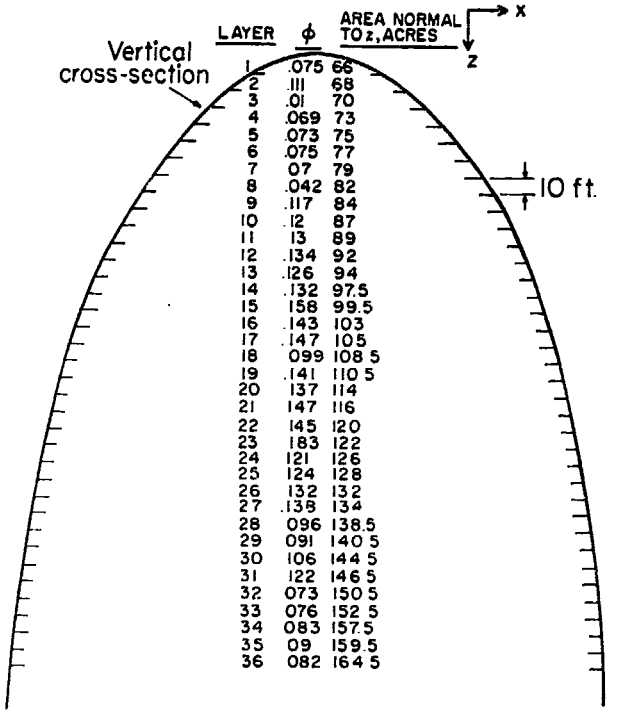

FIG. 2—POROSITY AND CROSS-SECTIONAL AREA OF A PINNACLE REEF.

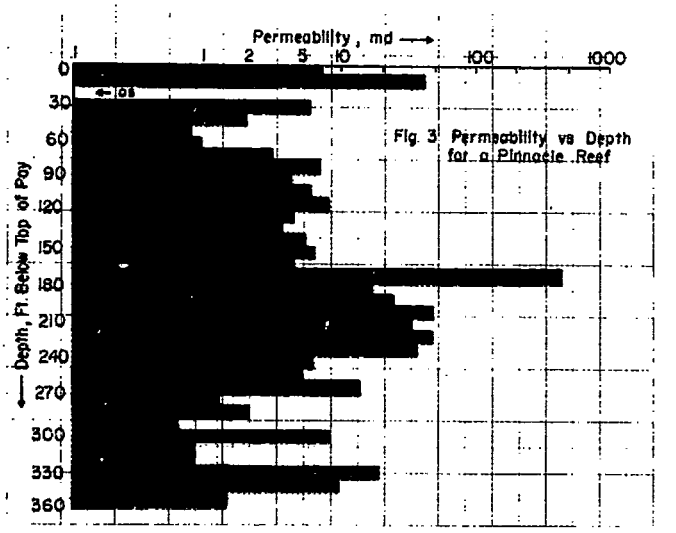

FIG. 3—PERMEABILITY VS DEPTH FOR A PINNACLE REEF.

initially was above its bubble point with no initial gas cap.

The three-dimensional, three-phase model was run in one dimension to perform the first three calculations. Run 1 was made using a constant, 2-day time step. At this time step nearly all blocks in the reservoir flowed less gas than they held and the method of handling gas percolation was not invoked. The solid curve of Fig. 4 shows calculated gas saturation vs depth after 3 years of production for this case of a 2-day Δt. The logarithmic scale on saturation is employed in order to show more clearly the small gas saturations in the oil column below the gas cap which develops at the top of the reservoir. This scale, however, conceals the significant variation in gas saturation near the top of the reservoir; this variation reflects the reservoir heterogeneity. For example, Fig. 4 shows that at the end of 3 years of production, gas saturation varies from about 29 percent in the fourth layer to about 14 percent in the sixth layer to 24 percent in the eighth layer. Since water saturation is 5 percent, the oil saturations in Layers 4, 6 and 8 are 66, 81 and 71 percent, respectively. These oil saturations correlate qualitatively with the respective layer permeabilities of about 6, 0.7 and 3 md. Oil will drain out of lower permeability blocks more slowly and, as a result, oil saturation at any time tends to be inversely proportional to layer permeability.

Fig. 5 shows oil pressure as a function of depth calculated from Run 1 using the 2-day time step. At this time of 3 years, the bottom of the developing gas cap is at a depth of about 100 ft from the top of the pay. Below this depth the pressure gradient is about the same as oil

TABLE 1 — SATURATION DEPENDENT QUANTITIES

Water Saturation	$P_{c(w-o)}$	k_{rw}	k_{rH}	$S_o/(1-S_w)$	$P_{c(g-o)}$	$\bar{k}_{ro}$	k_{rg}
.05	4	0	1.0	.02	4	0	.888
.065		.001	.9	.08		0	.888
.07	L	.004	.8	.105		0	.888
.10	I	.018	.49	.15		0	.842
.2	N	.06	.143	.2		.00001	.79
.3	E	.105	.04	.3	L	.00005	.675
.4	A	.147	.01	.4	I	.00033	.55
.5	R	.19	.00175	.5	N	.002	.425
.61		.266	0	.6	E	.0095	.315
1.0	0	1.0	0	.65	A	.018	.262
				.7	R	.032	.211
				.75		.057	.165
				.8		.098	.122
				.85		.17	.085
				.9		.298	.051
				.95		.546	.022
				1.0	0	1.0	0

NOTE: $k_{ro} = \bar{k}_{ro} \times k_{rH}$
The values $S_o/(1-S_w)$ are interpreted as total liquid saturation in obtaining k_{rg}; i.e., k_{rg} is a function of gas saturation only.

TABLE 2 — PRESSURE-DEPENDENT QUANTITIES

p, psia	B_w, RB/STB	B_o, RB/STB	B_g, RB/MCF	R_s, Mcf/STB	μ_o, cp	μ_g, cp
500	1.00000	1.141	5.895	.18	.886	.0131
700	.99860	1.165	4.1495	.217	.823	.01355
900	.99720	1.1899	3.1878	.26	.768	.0141
1300	.99440	1.234	2.1549	.352	.677	.0156
1644	.99230	1.2721	1.674	.434	.596	.0171
1800	.99090	1.2698		.434	.607	
2000	.98950	1.2668			.619	
2200	.98810	1.265			.632	
2400	.98670	1.262			.644	
2700	.98460	1.258			.662	
3000	.98250	1.2528			.683	
3300	.98040	1.249			.697	
3500	.97900	1.246			.704	

hydrostatic gradient. The gradient is slightly less than hydrostatic due to the small viscous gradient caused by the downward flow of oil toward the producing well interval in Layer 36.

Run 2 was performed using a 30.5-day Δt at which only about 10 of the 36 blocks held as much gas as they flowed. Thus the method described above was employed for 26 of the 36 blocks. The calculated saturation and pressure distributions for this 30.5-day Δt are compared with those using the 2-day Δt on Figs. 4 and 5. These figures show good agreement. Runs 1 and 2 resulted in virtually equal gas saturations in the developing gas cap in the top 100 ft of pay, except for the 10th layer where a 2.5 percent discrepancy occurred. In the oil column below the gas cap (below 100 ft), the 30.5-day time step using the method of handling gas percolation gave gas saturations less than 0.1 percent different from those calculated using the 2-day time step. The calculations did not converge at this 30.5-day time step when the method of handling the gas percolation was not employed.

A constant time step of 61 days was used in Run 3. Fig. 6 compares the calculated oil saturation distribution at a time of 16 years with

TABLE 3 — ADDITIONAL DATA

$\rho_w = \frac{70.44}{B_w}$ lbs_m/cu ft

$\rho_o = [51.84 + 64.5/(5.61416\ R_s)]/B_o$ lbs_m/cu ft

$\rho_g = 64.5/(5.61416\ B_g)$ lbs_m/cu ft

Initial oil pressure = 1700 psia 5 feet from top of pay

Initial gas saturation is zero everywhere

Initial water saturation is .05 everywhere

Production rate = 760 STB/D from layer 36

Number of grid blocks in z direction = 36

Block thickness = 10 feet

Initial oil-in-place = 26 million STB

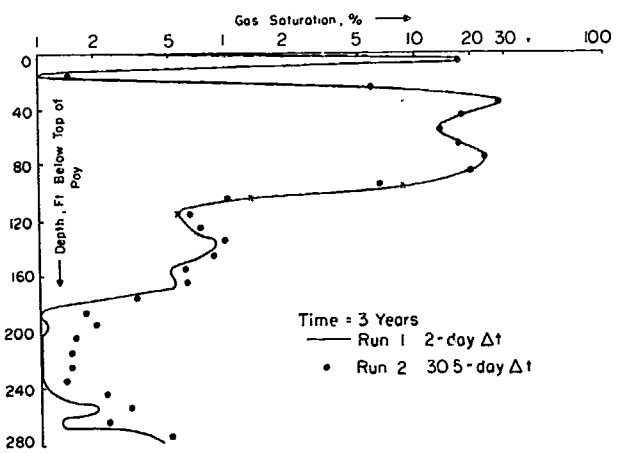

FIG. 4 — COMPARISON OF GAS SATURATION DISTRIBUTIONS FROM ONE-DIMENSIONAL CALCULATIONS.

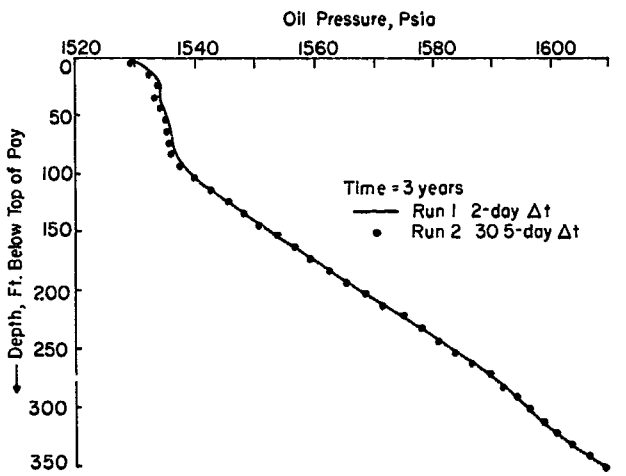

FIG. 5 — COMPARISON OF OIL PRESSURE DISTRIBUTIONS FROM ONE-DIMENSIONAL CALCULATIONS.

the results from Run 2 using the 30.5-day Δt. Again, agreement is very good.

A 30-year simulation of this bioherm using the 61-day Δt required 40 seconds of computing time on the CDC 6600 computer. Use of the 30.5-day Δt doubled this time while use of the 2-day Δt would have required about 20 minutes.

TWO- AND THREE-DIMENSIONAL RUNS

A two-dimensional calculation was performed for a reservoir 2,500 ft sq (areally) by 300 ft thick. An x-z (vertical slice) grid of 5 × 30 was employed so that each grid block was 500 × 2,500 × 10 ft. Each layer had a different permeability and porosity as listed in Fig. 2. Production rate was 760 B/D from the center block in the 27th layer. Pressure and saturation data and initial conditions were identical to those employed for Runs 1 through 3. Initial oil in place was 28.2 million STB.

A 5-year calculation was performed using a constant 30-day Δt. The calculated saturation distribution is difficult to represent in a simple plot because of the saturation reversals reflecting heterogeneity. Therefore, the gas saturations are simply given for each block on Fig. 7 at the end of 5 years. These saturations show the tendency of the gas to nose down toward the well in the center column where the well is completed. The sixty 30-day time steps for this 150-block system required 85 seconds at 6600 computer time.

It is interesting to note the ratio of vertical to horizontal transmissibility in this two-dimensional run. Computational difficulty in numerical simulation of reservoir performance generally increases with increasing values of this ratio. For the block dimensions here this ratio is the order of $(\Delta x/\Delta z)^2 = 2{,}500$. In spite of this high ratio, no computational difficulty was encountered using the 30-day time step.

A three-dimensional calculation was performed for this 2,500 × 2,500 × 300-ft reservoir using a grid of 5 × 5 × 30; thus, each block had dimensions of 500 × 500 × 10 ft. A 30-day time step was used for the first 4 years of the run and a 60-day step for 2 more years. Total computing time was about 10 minutes on the CDC 6600 computer. Assuming the continued use of a 60-day time step, this time extrapolates to 24 minutes total 6600 computer time for a 20-year simulation of this 750-block system. At a computer cost of $1,000 per hour, a three-dimensional simulation would cost about $400 per run. The time of 20 years corresponds to recovery of about 20 percent of the initial oil in place.

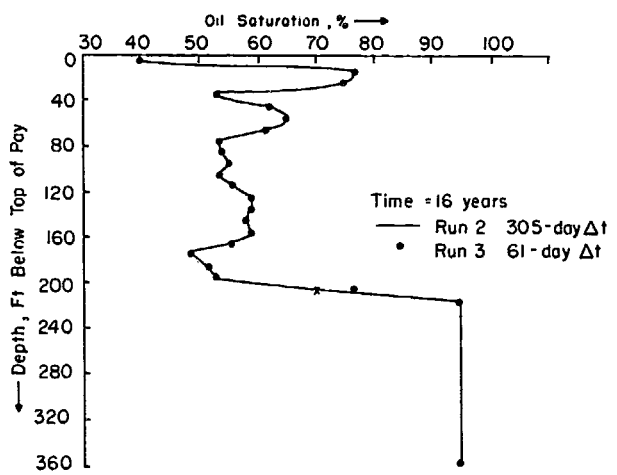

FIG. 6 — COMPARISON OF OIL SATURATION DISTRIBUTIONS FROM ONE-DIMENSIONAL CALCULATIONS.

CONCLUSIONS

The use of a reasonably sized time step in simulation of natural depletion often requires some method for handling a high-rate percolation of evolved gas upwards through the oil column.

A technique for handling this percolation has been developed and is easily integrated into a general multiphase, multidimensional calculation. The technique involves an assumption and is automatically applied to only those portions of the oil column which tend to satisfy the assumption. The technique allows use of considerably larger time steps in simulation of natural depletion with resultant reduction in computing expense.

The validity of the method has been examined by comparing saturation and pressure distributions that were calculated using a small time step, where the technique is not needed, to distributions that were calculated using a large time step where the technique must be employed. These comparisons have shown good agreement in a number of reservoir

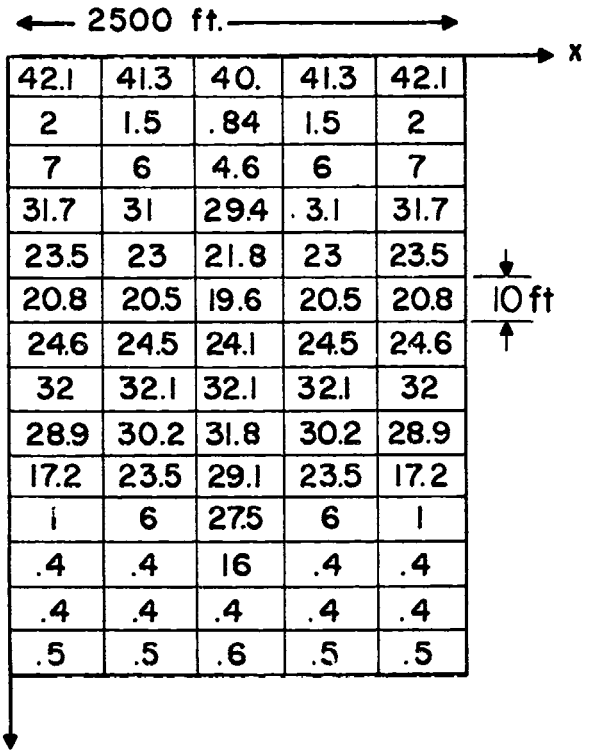

FIG. 7 — GAS SATURATION DISTRIBUTION AT 5 YEARS, FROM TWO-DIMENSIONAL CALCULATION.

studies, one of which is reported.

One-, two- and three-dimensional calculations of depletion of an example pinnacle reef reservoir were performed using the technique. Required CDC 6600 computer times were about 40 seconds per run for a 36-block, one-dimensional calculation and 24 minutes for a 750-block, three-dimensional calculation.

NOMENCLATURE

B = formation volume factor, reservoir volume/standard volume

b = formation volume factor, 1/B

k = permeability, md, or grid index for z direction

k_r = relative permeability

k_{rH} = hydrocarbon relative permeability

p = pressure, psia

$P_{c(w\text{-}o)}$ = water-oil capillary pressure, $p_o - p_w$, psi

$P_{c(g\text{-}o)}$ = gas-oil capillary pressure, $p_g - p_o$, psi

q_g = gas flow rate, Mcf/(sq ft-day)

S = fluid saturation, fraction

Δt = time increment, days

V_p = block pore volume, bbl

x,y,z = spatial variables, ft (z is vertical, or nearly so)

Δz = grid block dimension in z-direction

μ = viscosity, cp

ρ = density, lb_m/cu ft

γ = specific weight, $\rho g/(144 g_c)$, psi/ft

Φ = potential for use in Darcy's law[2], $\left(u_x = -\frac{k\gamma}{\mu}\frac{\partial \Phi}{\partial x}\right)$ where $\Phi = \int^p \frac{dp}{\gamma} - D$ and D is the depth measured vertically downward

SUBSCRIPTS

w = water

o = oil

g = gas

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