Introduction

This paper deals with the problem of estimating anisotropic parameters for depth migration over a transversely isotropic medium (TI). We present a wave equation migration velocity analysis (WEMVA) method based upon the differential semblance misfit function and elastic reverse time migration (ERTM). The method is an extension of the method for isotropic velocity analysis presented in Weibull and Arntsen (2011). The anisotropic parameters are estimated simultaneously through a iterative non-linear process aiming at minimizing the errors in the kinematics of the depth migrated images. In general, the velocity parameters can not be obtained uniquely from surface seismic data alone due to the lack of sensitivity and/or ambiguity and tradeoff between the different parameters (Grechka et al., 2002). Nevertheless, our method can be used to generate an accurate image of the subsurface that can be used as frame to draw better constrained and more unique solutions to the parameter estimation problem.

In the next section, we present the basic equations needed to set up and solve the optimization problem and then show two numerical examples which confirm the viability of the method in synthetic data.

Method and Theory

The theory for elastic reverse time migration is founded on non-linear inversion theory (Tarantola, 2005). Depth images are produced by crosscorrelating a source wavefield forward propagated in time with a residual wavefield backward extrapolated in time. In the context of elastic full waveform inversion, these images represent the gradients of the least square misfit function with respect to the material parameters. On the other hand, if the residual wavefield is given by the single scattering recorded data, we obtain Claerbout's imaging condition (Claerbout, 1971). According to this condition, given an accurate estimate of the material velocities, the crosscorrelation of the reconstructed source and receiver wavefields will have a maximum at zero lag in time and space. In Differential Semblance optimization we explore this fact to set up a non-linear least squares inversion problem. By parametrizing the image with an additional lag parameter we can capture the deviation of the maximum in crosscorrelation from zero lag, and use this to quantify the error in the estimates of the velocities.

In this paper we use an ERTM image parametrized by horizontal spatial lag ($\mathbf{h}$):

$$R(\mathbf{x},\mathbf{h}) = \sum_s \int_0^T dt\, \frac{\partial u_i^{fw}}{\partial x_i}(\mathbf{x}-\mathbf{h},t,s)\frac{\partial u_k^{bw}}{\partial x_k}(\mathbf{x}+\mathbf{h},t,s), \tag{1}$$

with Einstein summation convention over i and j. The wavefields u_i^{fw} and u_i^{bw} are the reconstructed source and receiver wavefields, respectively. These wavefields are computed by solving the constant density elastic wave equation:

$$\begin{aligned} u_i^{fw}(\mathbf{x},t,s) &= \int d\mathbf{x}' \int_0^T dt\, G_{ij}(\mathbf{x},t;\mathbf{x}',0) \sum_{sou=1}^{Nsou} \delta(\mathbf{x}'-\mathbf{x}_{sou}) \frac{\partial S}{\partial x_j}(\mathbf{x}_{sou},t,s) \\ u_i^{bw}(\mathbf{x},t,s) &= \int d\mathbf{x}' \int_0^T dt\, G_{ij}(\mathbf{x},0;\mathbf{x}',t) \sum_{rec=1}^{Nrec} \delta(\mathbf{x}'-\mathbf{x}_{rec}) \frac{\partial P^{rec}}{\partial x_j}(\mathbf{x}_{rec},-t,s) \end{aligned}$$

Where G_{ij} is the constant density elastic Green's function, S is the pressure source function, P^{rec} is the recorded pressure data, $\mathbf{x} = (x_1, x_3)$ are the spatial coordinates, $\mathbf{h} = (h_1, 0)$ is the subsurface horizontal half-offset, t is the time and s is the source index.

The Differential Semblace misfit function quantifies the deviation from zero lag, and is given by (Weibull and Arntsen, 2011):

$$DS = \frac{1}{2}\left\|\mathbf{h}\frac{\partial R}{\partial x_3}(\mathbf{x},\mathbf{h})\right\|^2 = \frac{1}{2}\int d\mathbf{x} \int d\mathbf{h}\, \mathbf{h}^2 \left[\frac{\partial R}{\partial x_3}(\mathbf{x},\mathbf{h})\right]^2, \tag{2}$$

The errors quantified by the Differential Semblance misfit function can be turned into velocity updates by a non-linear iterative optimization process. In this process, it is necessary to compute the gradients of the misfit function with respect to the velocity parameters.

The gradients can be computed in a similar fashion to the depth migration described above, by the adjoint state method (Chavent, 2009):

$$\nabla_{\mathbf{m}} DS(\mathbf{x}) = \sum_s \int dt \frac{\partial c_{ijkl}}{\partial \mathbf{m}}(\mathbf{x}) \frac{\partial u_l^{fw}}{\partial x_k}(\mathbf{x},t,s) \frac{\partial \psi_i^{fw}}{\partial x_j}(\mathbf{x},t,s)$$

$$+ \sum_s \int dt \frac{\partial c_{ijkl}}{\partial \mathbf{m}}(\mathbf{x}) \frac{\partial u_l^{bw}}{\partial x_k}(\mathbf{x},t,s) \frac{\partial \psi_i^{bw}}{\partial x_j}(\mathbf{x},t,s) \qquad (3)$$

Where c_{ijkl} is the elaticity tensor, and $\mathbf{m}$ depends on the velocity parametrization. In a 2D tranversely isotropic medium (TI), $\mathbf{m}$ consists of the P-wave velocity along the symmetry axis (V_0), the Thomsen's parameters (ε and δ; Thomsen (1986)), and the tilt angle of the symmetry axis (θ).

The wavefields ψ_i^{fw} and ψ_i^{bw} are adjoint states that can be computed by the following adjoint modelings:

$$\psi_i^{fw}(\mathbf{x},t,s) = \int d\mathbf{x}' \int_0^T \frac{\partial G_{ij}}{\partial x'_j}(\mathbf{x},0;\mathbf{x}',t) \int d\mathbf{h}\, \mathbf{h}^2 \frac{\partial^2 R}{\partial x'^2_3}(\mathbf{x}'+\mathbf{h},\mathbf{h}) \frac{\partial u_l^{bw}}{\partial x'_l}(\mathbf{x}'+\mathbf{2h},t) \qquad (4)$$

$$\psi_i^{bw}(\mathbf{x},t,s) = \int d\mathbf{x}' \int_0^T \frac{\partial G_{ij}}{\partial x'_j}(\mathbf{x},t;\mathbf{x}',0) \int d\mathbf{h}\, \mathbf{h}^2 \frac{\partial^2 R}{\partial x'^2_3}(\mathbf{x}'-\mathbf{h},\mathbf{h}) \frac{\partial u_l^{fw}}{\partial x'_l}(\mathbf{x}'-\mathbf{2h},t) \qquad (5)$$

Numerical examples

The first example in this section shows the behaviour of the Differential Semblance Misfit function for a simple TTI model. The model consists of a 1D layered model consisting of 3 layers with different values of the parameters V_0, ε, δ and θ, as shown in figures 1A-C. We simulate surface seismic data over this model with a maximum offset of 1400 m. To generate the data we use a finite difference solution to the elastic wave equation (Lisitsa and Vishnevskiy, 2010). Perturbing the magnitude of parameters in the second layer and computing the Differential Semblance error, one at a time, allows us to plot a 1D curve showing the variation of the Differential Semblance misfit function for each parameter, as shown in figures 1D-E. These plots show many interesting aspects of the Differential Semblance misfit function.

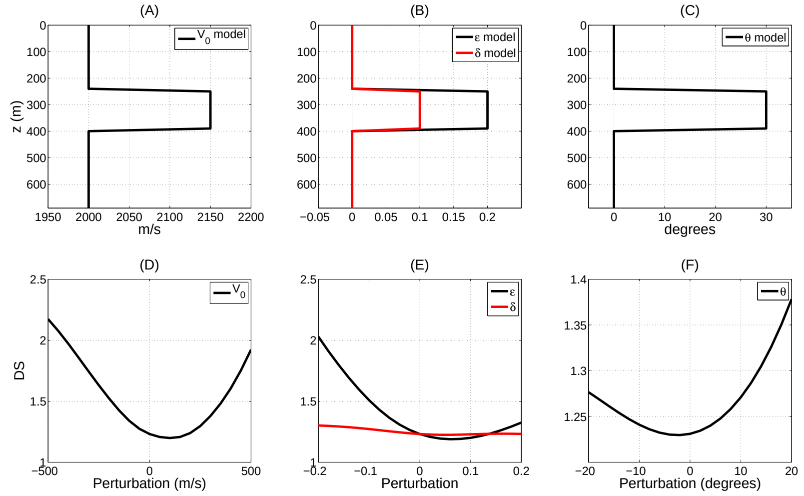

Figure 1 *A-C: 1D models used in example 1; D-F: Errors computed by perturbing the true models. Note that only one parameter is perturbed at a time, the other parameters are fixed at their true value.*

The major strength of this misfit function is that it is convex for a wide range of model perturbations, and therefore is particularly suitable for gradient based optimization. On the other hand a major weakness of the misfit function, is that the optimum velocity is not the true one. This means that to converge to a very accurate solution, additional constraints must be placed or other more refined misfit functions must be used, such as in full waveform inversion. The last comment is regarding the sensitivity of the Differential semblance with respect to the different parameters, which is a very dependent on the model and acquisition geometry. In this case, we see that the objective function is most sensitive to V_0, and ε.

In the second example we show the results of a Differential Semblance Optimization over a VTI synthetic model of the Gullfaks field off the Norwegian Margin. We attempt to simultaneously obtain estimates for V_0, ε and δ. The initial model is an isotropic model where V_0 is a 1D model linearly increasing in depth from 1480 m/s to 3200 m/s. The data simulates a streamer data with minimum offset of 150m and maximum offset of 6km. The maximum frequency in the data is 30Hz.

The results of the optimization pictured in Figure 2, show reasonable estimates of V_0, ε and δ. As can be seen from Figures 2B, 2C and 2D, the resolved parameters are strongly smoothed. This was a necessary constrain that helped reduce the null space and stabilize the optimization. At the same time, anisotropic parameters were constrained to have positive values only. The Common Image Point Gathers at $x = 2$ km, for the ERTM images constructed with the initial isotropic model, the optimized anisotropic model and the true set of parameters were ploted in Figure 3A. These angle gathers, show that the updated model succeeds in improving the kinematics of the migrated image, in particular for the deeper events. For completeness, Figure 3B shows a comparison of logs of the different parameters for the same position ($x = 2$ km).

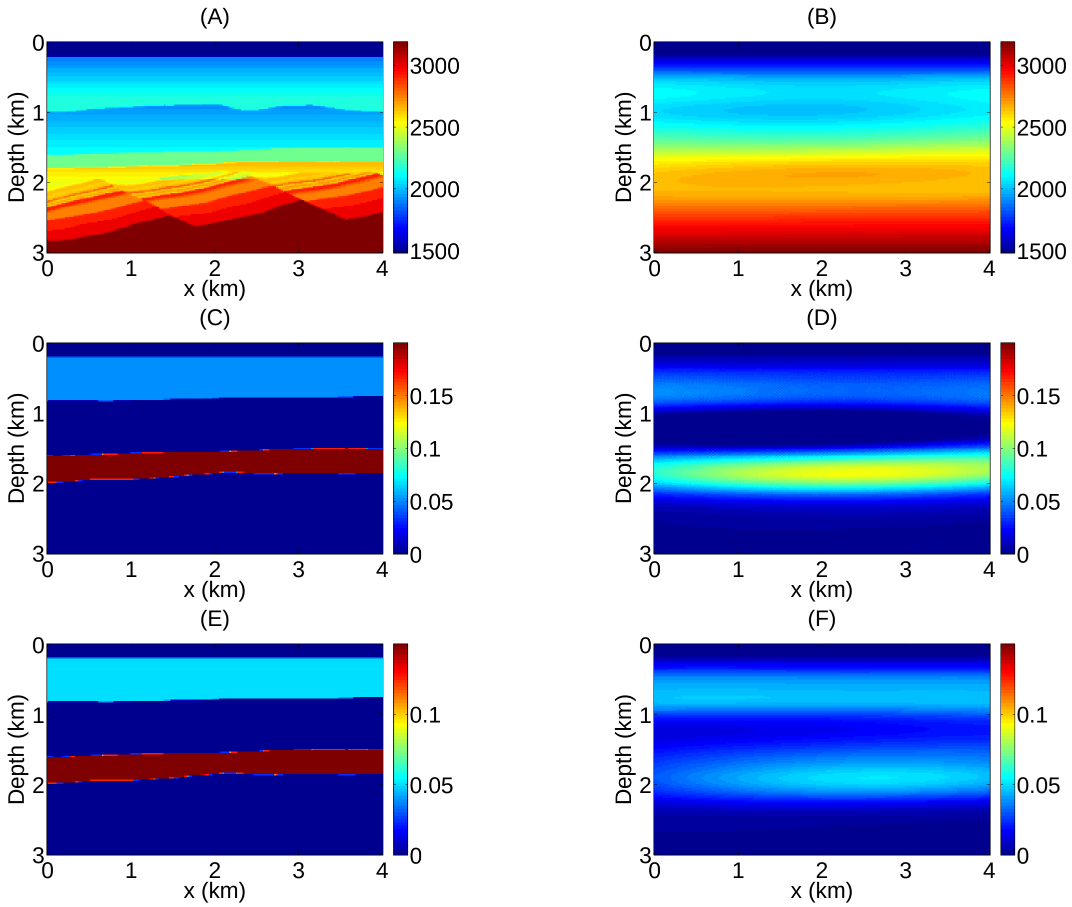

Figure 2 *A: True V_0 model; B: Updated V_o model; C: True ε model; D: Updated ε model; E: True δ model; F: Updated δ model.*

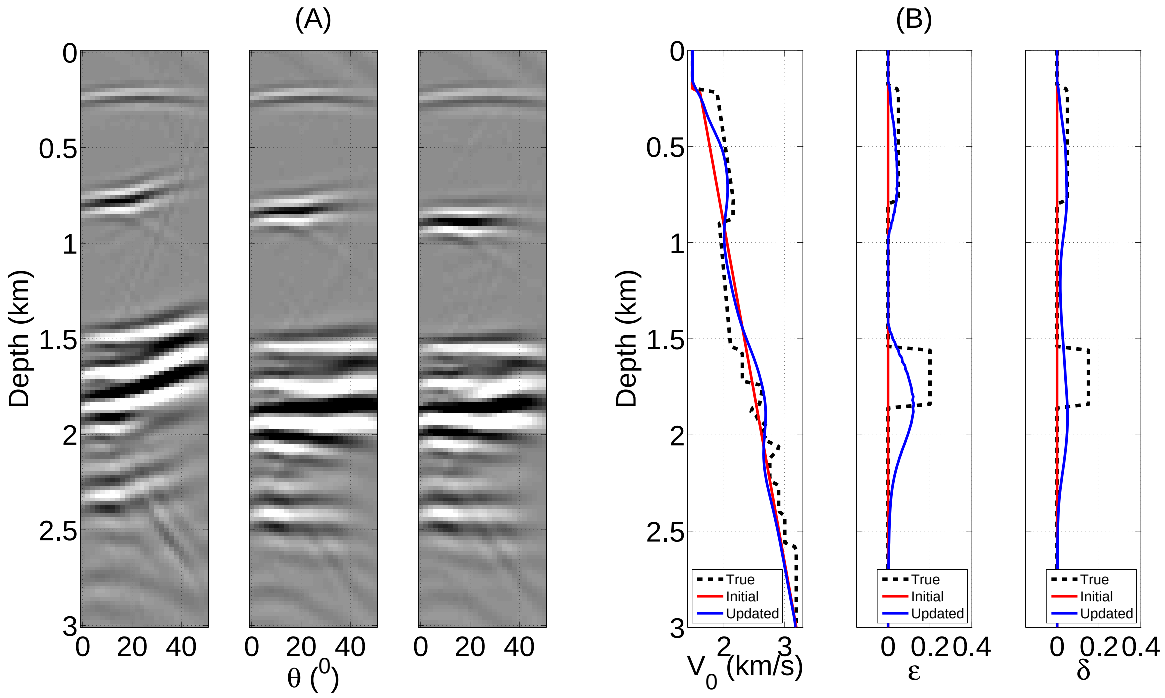

Figure 3 *A: Angle gathers - Initial (left), Updated(centre), True(Right); B: Comparison of velocity logs for V_0 (left), ε (centre), δ (Right).*

Conclusions

Anisotropic velocity models can be automatically estimated from surface seismic data by a non-linear optimization process based upon differential semblance and elastic reverse time migration. Through this process, the errors in the kinematics of migrated images are turned into parameter updates that help improve the positioning of the reflectors in the depth migrated image. This can be explored to create better constrained models and mitigate the inherent non-uniqueness of the solution of this type of inverse problem.

Acknowledgements

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