The influence of anisotropy on elastic full-waveform inversion
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SUMMARY

Elastic full-waveform inversion (FWI) is most commonly used for isotropic media, whereas anisotropic FWI is usually done in the acoustic approximation. We have compared different assumptions commonly used in FWI: elastic versus acoustic and isotropic versus anisotropic (VTI) using a dataset that is anisotropic and elastic. The results show that while the acoustic approximation works fairly well in this case, assuming isotropy does not. Ignoring anisotropy for FWI can lead to significant errors. However, we were able to recover the vertical P-wave velocity \( V_{P0} \) quite well even with poor models for the anisotropy, indicating that perfect knowledge of the subsurface anisotropy is not necessary in order to invert for \( V_{P0} \) and achieve good results. It is also shown that it is possible to invert for the anisotropy parameters in complex media.

INTRODUCTION

Full-waveform inversion (FWI) is a technique for estimating subsurface properties by using the entire recorded waveform and applying inverse theory (Tarantola (1984), Mora (1987)). Acoustic FWI can work well in cases where the elastic effects have a weak signature in the data (Brossier et al. (2002)). Elastic isotropic FWI has proved to yield reliable results for complex geologies (Shipp and Singh (2002)). For the time being, anisotropic FWI is still mostly performed using the acoustic approximation (Warner et al. (2013), Plessix et al. (2014)). Acoustic wave propagation is commonly used because of the much lower computational cost involved, compared to elastic FWI. However, for acoustic FWI to work, the data must contain diving waves and refractions, and hence long offset data are required (Virieux and Operto (2009)). For limited-offset data dominated by reflected waves, the elastic approximation of wave propagation is needed (Raknes and Arntsen (2014)).

Others have shown that the choice of parameterization is important for anisotropic FWI (Ghalami et al. (2013)). Alkhalifah and Plessix (2014) claim that parameterizations that involve the P-wave NMO velocity \( V_{nmo} \), anisotropy parameters \( \delta \) and \( \eta = (\varepsilon - \delta)/(1 + 2\delta) \), where \( \varepsilon \) and \( \delta \) are the Thomsen parameters (Thomsen (1986)); or P-wave horizontal velocity \( V_{hor} \), \( \eta \) and \( \varepsilon \) are good for acoustic FWI in VTI media.

It has also been shown that it is possible to recover Thomsen parameters in homogeneous media by inverting for the vertical S-wave velocity \( V_{S0} \), \( V_{P0} \), \( V_{nmo} \) and \( V_{hor} \) and then calculating \( \varepsilon \) and \( \delta \) from the results (Kamath and Tsvankin (2014)). In this study, we want to see if it is possible to use \( V_{P0} \), \( V_{S0} \), \( \varepsilon \) and \( \delta \) as parameters in anisotropic, elastic FWI. We investigate the influence of anisotropy on full-waveform inversion, comparing the acoustic and elastic approximations. We also invert directly for \( \varepsilon \) and \( \delta \) when \( V_{P0} \), \( V_{S0} \) and \( \rho \) are known, and show that they can be recovered from poor starting models in a more complicated model.

THEORY

For a comprehensive review of the theory behind FWI, see Virieux and Operto (2009). What is included here is a brief overview of the basics. In FWI we want to find a parameter model \( \mathbf{m} \) that can produce modeled data \( \mathbf{u} \) which is close to some measured data \( \mathbf{d} \). A basic assumption is that synthetic data \( \mathbf{u} \) can be generated numerically by solving some wave equation. If we let \( \mathcal{L} \) be a numerical wave operator that maps \( \mathbf{m} \) from the model domain into the data domain, we can generate synthetic data by applying \( \mathcal{L} \) on the model \( \mathbf{m} \):

\[
\mathcal{L} (\mathbf{m}) = \mathbf{u}.
\]

If there exists an inverse operator \( \mathcal{L}^{-1} \) that maps the data back into the model domain, then the solution is

\[
\mathbf{m} = \mathcal{L}^{-1} (\mathbf{d}).
\]

Finding this inverse operator, however, is in practice not possible. We therefore define a measure, \( \mathcal{F} (\mathbf{m}) \), between \( \mathbf{u} \) and \( \mathbf{d} \). This measure is often called the objective or misfit functional, and is here defined as follows (Raknes and Arntsen (2014)):

\[
\mathcal{F} (\mathbf{m}) = \frac{1}{2} \sum_{j=0}^{n_s} \sum_{i=0}^{n_r} \| \mathbf{u}_{i,j} (\mathbf{m}) - \mathbf{d}_{i,j} \|^2.
\]

where \( \mathbf{u}_{i,j} (\mathbf{m}) = \mathbf{u}_{i,j} (\mathbf{m})/\| \mathbf{u}_{i,j} (\mathbf{m}) \| \) is the normalized modeled data, \( \mathbf{d}_{i,j} = \mathbf{d}_{i,j}/\| \mathbf{d}_{i,j} \| \) is the normalized measured data, \( n_r \) is the number of receivers in the data set and \( n_s \) is the number of shots. \( \| \cdot \|_2 \) refers to the \( L^2 \)-norm.

We require the solution of the problem to be an extreme point of \( \mathcal{F} (\mathbf{m}) \). Hence, the solution can be written

\[
\mathbf{m}' = \arg \min_{\mathbf{m}} \mathcal{F} (\mathbf{m}),
\]

where \( \mathbf{m}' \) is the model we are searching for. The inverse problem is non-linear and ill-posed.

Trying to find extreme points of \( \mathcal{F} (\mathbf{m}) \) is done using an iterative optimization algorithm:

\[
\mathbf{m}_{k+1} = \mathbf{m}_k - \alpha_k H_k^{-1} g_k,
\]

where \( H_k \) is the Hessian of \( \mathcal{F} \) and \( \alpha_k \) is the step size.
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where $\alpha_k > 0$ is the step length, $H^{-1}$ is the inverse Hessian matrix and $g_k$ is the gradient of $F(m)$ with respect to $m$ at step $k$. An initial model $m_0$ is required to start the algorithm, and it is then run until some convergence criterion is fulfilled.

The crucial step in FWI is computing the gradient. Using the adjoint state method (Tarantola (1984), Mora (1987)), we can find the gradients needed for FWI. The elastic wave equation can be written in the following form:

$$\rho \ddot{u}_i - \partial_j c_{ijkl} \partial^j u_k = f_i,$$

$$c_{ijkl} \partial_j u_k n_j = T_i,$$

where $u_i = u_i(x_s, x, t)$ is the $i$th component of displacement resulting from a shot (i.e., body force $f_i$ and/or traction $T_i$) located at $x_s$, $c_{ijkl}$ is the stiffness tensor, $n_j$ is a normal vector and $\rho$ is the density.

Finding expressions for the gradients in a VTI medium is done by following the approach of Mora (1987) and assuming VTI instead of isotropy. He finds a solution for the displacement perturbation $\delta u_i$ using Green’s functions ($G_{ij}$) as

$$\delta u_i = \int_V dV \dot{G}_{ij} * \delta f_j + \int_S dS \dot{G}_{ij} * \delta T_j$$

where $\delta f_j$ and $\delta T_j$ are the perturbations of the body force and traction terms, respectively.

Using this equation to define the Frechet kernel and writing out the stiffness tensor $c_{ijkl}$ for VTI, we find expressions for the model update gradients in 3D:

$$- \int_V dV \dot{G}_{ij} * \delta \rho - \int_V dV (\partial G_{ij} * (\delta c_{jklm} u_{l,m})).$$

(7)

Figure 1: True model for a) $V_{p0}$ and b) $\varepsilon$.

Figure 2: a) Initial $V_{p0}$ model created using WEMVA. b) Initial model for $\varepsilon$. 

$$\delta \rho = - \sum_{n_s} \int dT u_j \Psi_j,$$

$$\delta c_{11} = - \sum_{n_s} \int dt (u_{1,1} + u_{2,2}) (\Psi_{1,1} + \Psi_{2,2}),$$

$$\delta c_{33} = - \sum_{n_s} \int dt u_{3,3} \Psi_{3,3},$$

$$\delta c_{13} = - \sum_{n_s} \int dt [\Psi_{3,3} (u_{1,1} + u_{2,2}) + (\Psi_{1,1} + \Psi_{2,2}) u_{3,3}],$$

$$\delta c_{44} = - \sum_{n_s} \int dt [(\Psi_{3,1} + \Psi_{1,3}) (u_{3,1} + u_{1,3}) + (\Psi_{3,2} + \Psi_{2,3}) (u_{3,2} + u_{2,3})],$$

$$\delta c_{66} = - \sum_{n_s} \int dt [(\Psi_{2,1} + \Psi_{1,2}) (u_{2,1} + u_{1,2})].$$

(8)
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\[-2(\Psi_{2,2u_{1,1}} + \Psi_{1,1u_{2,2}})\]

Here, \(u_{i,j}\) is the derivative in the \(j\)-direction of the forward modelled field in the \(i\)-direction. \(\Psi_{i,j}\) is the equivalent for the backward modelled field.

A change of variables gives the gradients in terms of \(V_{P0}\), \(V_{S0}\), \(\varepsilon\), \(\delta\) and \(\gamma\):

\[
\delta V_{P0} = 2\rho V_{P0}(2\varepsilon + 1)\delta c_{11} + 2\rho V_{P0}(V_{P0}^2 - V_{S0}^2) + 2\delta V_{P0}(2V_{P0}^2 - V_{S0}^2)\delta c_{13} + 2\rho V_{P0}\delta c_{33},
\]

\[
\delta V_{S0} = 2\rho \left[\frac{V_{S0}(V_{S0}^2 - V_{P0}^2) - \delta V_{P0}V_{S0}}{(V_{P0}^2 - V_{S0}^2)^2 + 2\delta V_{P0}(V_{P0}^2 - V_{S0}^2)}\right] - V_{S0}\delta c_{13} + 2\rho V_{S0}\delta c_{44} + 2\rho V_{S0}(2\gamma + 1)\delta c_{66},
\]

\[
\delta \varepsilon = 2\rho V_{P0}^2\delta c_{11},
\]

\[
\delta \delta = \rho \left[\frac{V_{P0}^2(V_{P0}^2 - V_{S0}^2)}{(V_{P0}^2 - V_{S0}^2)^2 + 2\delta V_{P0}(V_{P0}^2 - V_{S0}^2)}\right] \delta c_{13},
\]

\[
\delta \gamma = 2\rho V_{S0}^2\delta c_{66}.
\]

Finding the inverse Hessian matrix in eq. 5 is computationally difficult, as it contains second-derivatives of the objective function with respect to \(m\). Therefore, the minimization is done via the limited-memory Broyden-Fletcher-Goldfarb-Shanno (L-BFGS) algorithm (Nocedal and Wright (2006)), which is a quasi-Newton method that estimates the inverse Hessian matrix by using gradients from previous iterations.

RESULTS

We have used a fairly complex model (Figure 1), including realistic models for \(\varepsilon\) and \(\delta\), and performed several inversions of \(V_{P0}\) in order to compare the results from various assumptions (acoustic, elastic, isotropic, anisotropic) when there is significant anisotropy present in the data. For the acoustic tests, \(V_{S0}\) is set to zero, meaning no shear stress is present. \(V_{S0}\) and \(\rho\) are structurally similar to \(V_{P0}\), while \(\delta\) is structurally similar to \(\varepsilon\). The model is 2D, 10 km long, 3 km deep and representative of the Gullfaks field in the North Sea. We present results from FWI using this dataset and the following assumptions: 1) acoustic, isotropic 2) acoustic, anisotropic 3) elastic, isotropic 4) elastic, anisotropic.

For all tests we used a 10 m grid spacing on both axes, 100 m shot spacing and receivers in every grid point in the x-direction. This gives a total of 1001 receivers and 101 shots. Receivers and shots were positioned at 10 m depth. A Ricker wavelet with a center frequency of 5 Hz was used as source. There was no free surface included in the inversion. For each shot, 3.9 seconds of pressure data was recorded.

In all cases the initial model was the model shown in Figure 2a. Figure 3a shows the results of the inversion where we inverted for \(V_{P0}\) in the elastic case with every other parameter (\(\rho, V_{S0}, \varepsilon, \delta\)) known exactly. This will be the baseline for how well FWI can be expected to perform in our complex model with this survey geometry and frequency range. We see that the acoustic approximation (Figure 3b) works well in this case, due to the long offset data used in this study.
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Figure 5: Inverted models for $V_{P0}$ with $\epsilon$ and $\delta$ models equal zero, a) elastic and b) acoustic (elastic, $V_{S0} = 0$).

Another inversion was performed with severely smoothed versions of the $\epsilon$ and $\delta$ models (Figure 2b). These results are shown in Figure 4. It is clear that the results in the anisotropic top part of the model are not reproduced as well as earlier. Velocities inside the low-velocity layer are also inaccurate. However, the boundaries of the layer have been reconstructed fairly well, as have the deeper structures.

In Figures 5a and 5b we see the results of an inversion without taking into account anisotropy. We have inverted $V_{P0}$ with $\epsilon = \delta = 0$. This causes major overfitting artifacts all over the model. Both the elastic and the acoustic inversions fail in this case. FWI of an anisotropic dataset without taking the anisotropy into account will not work very well.

For the simultaneous $\epsilon$ and $\delta$ inversion, the starting model in Figure 2b was used for $\epsilon$. For $V_{P0}$, $V_{S0}$, and $\rho$, the exact models were used. As seen in Figure 6, $\epsilon$ has been better recovered than $\delta$. For $\epsilon$, the deepest anisotropic layer has been almost fully recovered. However, there are side lobes with small negative values all around this layer. The shallow anisotropic layer has also been recovered very well, both in value and physical location. The $\delta$ inversion has recovered the boundaries of the anisotropic layers, but has not been able to fully recover the correct magnitudes, due to $\epsilon$ having more of an impact on the the long offset data than $\delta$.

CONCLUSIONS

In this study we have applied four different strategies to inversion of $V_{P0}$ using a synthetic elastic and anisotropic dataset, as well as shown that it is possible to invert for $\epsilon$ and $\delta$ at the same time in a complex model when $V_{P0}$, $V_{S0}$ and $\rho$ are known. It is clear that the acoustic approximation holds well when inverting for $V_{P0}$ with long offset data. We also see that neglecting anisotropy is a critical mistake. The inversions where anisotropy was not taken into account failed. However, the inversion using the smooth anisotropy model gave fairly good results. This shows that one does not need a perfect model of the subsurface anisotropy in order to perform elastic FWI in anisotropic media, but some knowledge is necessary.

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